Analytical procedure to obtain internal parameters from performance curves of commercial thermoelectric modules

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ABSTRACT

Manufacturers of commercial thermoelectric modules provide datasheets of the modules including information and graphs about the performance attained at several working conditions. Details about internal parameters are not made available to customers, because in the broad majority of the cases they are not necessary. However, when developing non-standard applications or conducting research projects it is sometimes necessary to make the modules work in different conditions than those shown in the performance curves. This paper shows a methodology to extract thermoelectric internal parameters from the information provided by performance curves, hence allowing scientists to predict the performance of the module at any working condition. The method is based on the basic equations that link thermal and electrical dynamics in which some parameters must be estimated. As a result it is possible to predict the behavior of the modules if they are operated in a non-standard way. One good example is to simulate how a module designed for cooling applications will behave if used as a Seebeck module for power generation. The proposed methodology has been successfully applied to a commercial Peltier module for which the behavior as a thermoelectric generator was simulated and then tested experimentally, attaining very similar results.

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1. Introduction

Nowadays, thermoelectric modules are so specialized that every manufacturer offers a set of modules for each kind of application. Modules within the same series are intended to be used in the same kind of application, and the set comprises modules with diverse external dimensions, while separate series differ mostly in the internal parameters such as number of pellets, pellet dimensions and geometry, and assembling procedure. The information contained in the datasheets of themoelectric modules mainly describes external dimensions and maximum performance values. Each family of TE modules is optimized in performance and manufacturing costs for a given application: cooling applications, power generation at low temperature, power generation at high temperature, etc. For a standard application it is not difficult to select the correct module looking at the performance values of the appropriate family of modules, then it is possible to estimate the final behavior of the application by using the performance curves (provided that the temperatures can be estimated accurately).

However, for non-standard applications it can be necessary to make the modules work at an atypical condition, graphically located outside of the boundaries of the performance curves. In this case it is very helpful to use an analytical model able to predict the behavior of the module. An extreme example is to use a Peltier module, originally designed for cooling applications, to generate electricity as if it were a TEG or Seebeck module. A good reason for such an abnormal use of a TE module is to obtain electrical power at higher voltages that with standard TEG, hence avoiding DC–DC amplifiers in some applications. Most TEG modules are designed for low thermal resistance by using big active elements with small height, hence allowing for higher heat transfers through the module which yields to higher electrical power. Nevertheless, this design also produces lower output voltage that, for some applications, may require a DC–DC amplifier or mounting several modules connected in electrical series. On the other hand, Peltier modules are typically designed for 12 V operation and use more pellets with larger height, as a consequence if they are used for power generation, they produce lower electrical power but higher voltage than TEG modules operated at the same conditions. As a summary it can be said that Seebeck modules generate mV at low temperatures while Peltier modules may generate volts at the same operating conditions. Due to increasing number of portable devices (including phones, mp3 players, and digital cameras) with the associated problem of eventually running out of batteries, there is active research in the field of low-power electric generation from sources such as movement, heat or sun. Thermoelectricity plays an important role in the conversion of heat into

electricity. However, since Seebeck modules are typically designed for temperatures in the range of 250 °C, they do not perform well in domestic or office environment where potential heat sources for producing electricity have lower temperatures. On the other hand, internal design of Peltier modules is more appropriate for this kind of application if used as thermoelectric generator, but performance curves of Peltier modules operating as thermoelectric generators are not provided.

In order to obtain the internal parameters of a given module, there are two possible approaches: obtaining the properties experimentally, or estimating parameters from the information available in the vendor datasheet. From the small number of publications related to this topic, the most significant are the experimental procedure described by Huang [6] and the mathematical methods described by Luo [5].

Obtaining the parameters experimentally is interesting, especially before building a prototype application, because the parameters obtained represent the actual behavior of the module in a working condition which is very close to that of the final application. Nevertheless, the experimental approach is not useful for selecting a module from the broad vendor's catalogs during the design stage. In contrast, the mathematical approach based on information available on product datasheets is more valuable for designing applications. Luo proposed two methods for computing internal parameters based on the values V_{max}, I_{max}, Q_{max} and ΔT_{max}, available in datasheets. Since those four values are obtained at different working conditions (and also at extreme operation) they are inconsistent and do not match the basic thermoelectric equations altogether. Hence, two methods are proposed in [5]: method I uses V_{max}, I_{max}, and ΔT_{max} while method II uses I_{max}, Q_{max} and ΔT_{max}, obtaining different sets of parameters with up to 5% discrepancy in the modules analyzed.

The procedure presented in this paper falls in the category of analytical methods, but instead of using three single outmost values it uses several points and minimizes the error by linear regression or curve fitting. In the proposed procedure many values are extracted form the performance graphs provided by the manufacturer in typical datasheets, in contrast with just two or three extreme numerical values used in the method proposed in [5]. Although significantly more complicated to implement, the procedure presented in this paper is more accurate and can be used to simulate the behavior in a wider range of working conditions, including the simulation of Peltier modules working as thermoelectric generators.

2. Analytical procedure to obtain Peltier parameters

Beginning with a module typically designed for cooling applications, the performance graphs provided by the manufacturer show the maximum temperature difference (ΔT) for different values of current supplied and heat transfer. However, it is not possible to know from these curves how much electrical energy could be obtained if the module is used as an electric generator for a given heat flow. The proposed method makes use of the basic thermoelectric equations [1] to estimate internal parameter (α, β, and γ) that can be used subsequently to estimate the behavior in any working condition, including the behavior as electric generator.

2.1. Basic Peltier equations

If a thermoelectric module is supplied with a direct current I (A), a certain amount of heat Q_c (W) is absorbed in the cold side and pumped to the hot side. Because of the Peltier effect the heat pumped is proportional to the value of the current and the temperature at the cold side T_c (K), being α (V/K) the Peltier coefficient.

However, in a real thermoelectric module, the heat absorbed in the cold side is not just the Peltier term, because it is diminished by the power due to electrical resistance and heat transfer due to thermal conductivity.

As a consequence of the electrical current going through the module there is heat generated internally (Joule effect), which is proportional to the electrical resistance ρE (Ω) and the square of the current. The power generated by Joule effect is considered to heat the hot and cold sinks in the same amount. The last term in the equation for computing the heat at the cold sink (Eq. (1)) [1] computes the amount of heat transferred from the hot sink to the cold sink due to internal thermal resistance. In this case the amount of power is proportional to the conductivity λ (W/m/K) and the temperature difference $\Delta T = T_h - T_c$ (K).

Assuming that internal material properties (α and λ) are constants that do not depend on the temperature, the equation for the heat power taken at the cold sink is:

$$Q_c = \alpha I T_c - \frac{1}{2}\rho E I^2 - \frac{\lambda \Delta T}{E} \tag{1}$$

where E (m/m^2) is a geometric property of the pellets defined as height divided by the cross section. Defining $\gamma = \lambda/E$ (W/K) (thermal conductance) and $\beta = \rho E$ (Ω) (electrical resistance) the following equation for a given value of E is obtained:

$$Q_c = \alpha I T_c - \frac{1}{2}\beta I^2 - \gamma \Delta T \tag{2}$$

Similar equations can be used for the heat power given at the hot sink, where only the sign of the middle term is changed from negative to positive:

$$Q_h = \alpha I T_h + \frac{1}{2}\rho E I^2 - \frac{\lambda \Delta T}{E} \tag{3}$$

$$Q_h = \alpha I T_h + \frac{1}{2}\beta I^2 - \gamma \Delta T \tag{4}$$

The electrical power supplied to the system is simply the product of the current and voltage. These magnitudes are not linked by a constant resistor value like in other electronic circuits, because for a given current the required voltage depends on the temperature of the system given that ρ depends on the temperature.

$$W = V \cdot I \tag{5}$$

Hence the power balance equation will be given by Eqs. (2), (4), and (5).

$$Q_h - Q_c = W$$
$$\alpha I T_h + \frac{1}{2}\beta I^2 - \gamma \Delta T - \alpha I T_c + \frac{1}{2}\beta I^2 + \gamma \Delta T = V \cdot I \tag{6}$$
$$\alpha(T_h - T_c)I + \beta I^2 = V \cdot I$$

Therefore the following simplified equation is obtained:

$$V = \alpha \Delta T + \beta I \tag{7}$$

The important advantage of Eq. (7) is that it contains just two parameters that can be easily estimated from the performance curves provided by the manufacturer. After obtaining α and β, the other properties γ, λ and ρ can be derived directly.

2.2. Estimating parameters from performance curves

In this section we take as an example the performance curves of the thermoelectric module Marlow DT 12-6 which are shown in Fig. 1. This particular module has 127 thermoelements connected in series in module measuring 40 × 40 mm designed for working temperatures up to 150 °C.

It can be seen that two sets of curves are provided, one set for T_c = 27 °C (T_c = 300 K) and another for T_c = 50 °C (T_c = 323 K). On
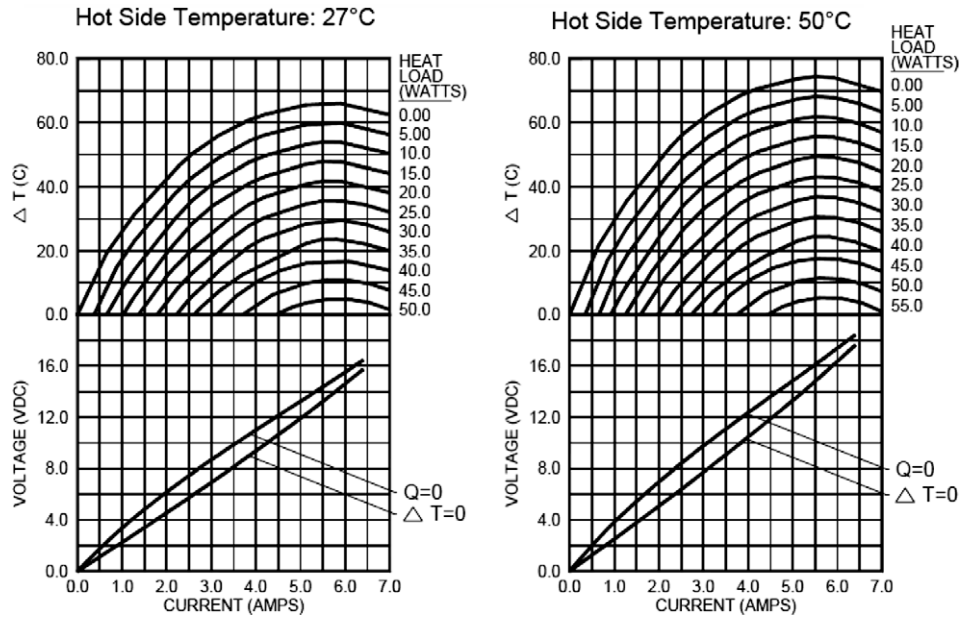
Fig. 1. Performance curves of Marlow DT 12-6 [2].

each set there are curves to obtain ΔT as a function of the electrical current for different values of Q_c, and also curves to obtain required voltage as a function of the current. In the later case only two extreme situations are represented $\Delta T = 0$ and $Q_c = 0$, since all other situations lay in between these curves. Applying Eq. (7), inferred in the previous section, to the lower set of graphs the value of β can be obtained directly. Assuming $\Delta T = 0$, Eq. (7) becomes $V = \beta I$, therefore $\beta = \frac{V}{I}$ which is the slope of the curve $\Delta T = 0$ (quite a straight line since the internal resistance is constant for constant temperatures). In the example being shown, the following values are obtained for β as extracted from the graphs are $\beta = 2.431 \ \Omega$ for $T_h = 27 \ °C$ and $\beta = 2.769 \ \Omega$ for $T_h = 50 \ °C$.

The next step in the proposed procedure is aimed to obtain the value of parameter α, which is the parameter directly related to the thermoelectric effect. Knowing the value of β it is possible to apply Eq. (2) also for $\Delta T = 0$, hence

$$Q_c = \alpha I T_c - \frac{1}{2}\beta I^2 \tag{8}$$

So the value of α will be obtained with the following expression:

$$\alpha = \frac{Q_c + \frac{1}{2}\beta I^2}{I T_c} \tag{9}$$

By setting $\Delta T = 0$ we have $T_c = T_h$, and the value of β is already known, so Eq. (9) has Q_c and I as the only independent variables. Consequently, the value of α can be obtained with any pair of values (Q_c, I) obtained from the upper set of curves for $\Delta T = 0$ (this is at the x-axis). Instead of using Q_{max} and I_{max}, our suggestion is to compute α as the average of several pair of values (Q_c, I) to minimize the error. Note that Q_c and I are not linearly related in Eq. (9) so the curves for constant Q do not cut the axis evenly spaced.

Finally, the last characteristic of the module materials is the parameter γ, related with the thermal conductivity. In this case, Eq. (2) for $Q_c = 0$ will be used.

$$\gamma = \frac{\alpha I T_c - \frac{1}{2}\beta I^2}{\Delta T} \tag{10}$$

Therefore, the value of γ can be obtained with any pair of values $(I, \Delta T)$ extracted from the curve corresponding to $Q_c = 0$ in the upper graphs of Fig. 1. Again, it is recommended to average several estimations of γ obtained with different pairs of I and ΔT.

The values of α, β and γ obtained for the Thermoelectric module Marlow DT 12-6, used as an example are shown in Table 1.

The method to estimate these parameters based on I_{max}, V_{max}, Q_{max} and ΔT_{max} [6] differ from these values 18.5% in the worst case. The main reason for a better accuracy of the method presented in this paper is the use of several data point read from the datasheet graphs instead of using just the values corresponding to the most extreme condition.

2.3. Simulating Peltier behavior

Once the internal parameters of the module have been obtained, it is possible to analytically simulate the behavior of the module at any working condition by using the basic equations described in Section 2.1. For example, Eq. (2) can be rewritten to obtain ΔT as a function of I as:

$$\Delta T = \frac{\alpha I T_h - Q_c - \frac{1}{2}\beta I^2}{\alpha I + \gamma} \tag{11}$$

Using Eq. (11) for $T_h = 27 \ °C$ and $Q_c = 10 \ W$, $20 \ W$ and $30 \ W$, three of the characteristic curves for constant heat power are obtained. Fig. 2 includes dots representing the values simulated using Eq. (11), over the experimental curves from the manufacturer datasheet. This graph shows good precision of the simulated data and therefore confirms the accuracy of the proposed parameter estimation procedure.

Simulation within the space of working conditions used for estimating the internal parameters is expected to be accurate after using any system identification procedure. The accuracy while simulating new working conditions is difficult to predict, but assuming the correctness of the basic thermoelectric equations and using precise parameters such simulations are expected to

Table 1
Internal parameters determined from performance curves.

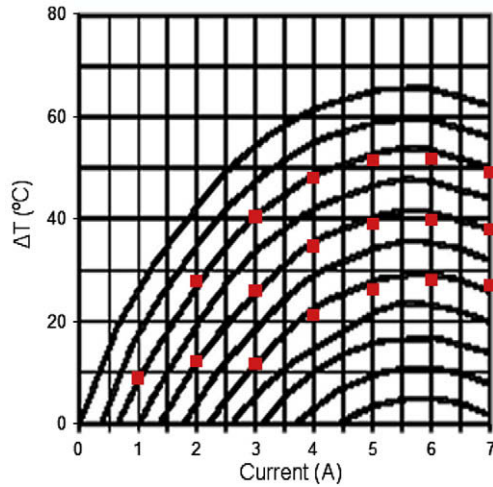|	Th = 27 °C (300 K)	Th = 50 °C (323 K)
α (V/K)	0.0543	0.055
β (Ω)	2.431	2.769
γ (W/K)	0.528	0.494

Fig. 2. Simulated dots in a layer over the original performance curves of Marlow DT 12-6 [2].

be acceptable. The following section describes the process to simulate probably the most extreme situation, which is the behavior of the Peltier module being used as a thermoelectric generator.

3. Simulating Seebeck performance

After obtaining the internal parameters of the module, it is possible to apply the fundamental equations to predict its behavior under different working conditions. It is even possible to simulate the behavior of a Peltier module working as a Seebeck electrical generator since both Peltier and Seebeck equations share the same internal parameters.

3.1. Basic Seebeck equations

In the case of Seebeck effect, Eqs. (1) and (2) become the following expressions in which the signs of the last two terms have changed [1]:

$$Q_c = \alpha I T_c + \frac{1}{2}\rho E I^2 + \frac{\lambda \Delta T}{E} \tag{12}$$

$$Q_c = \alpha I T_c + \frac{1}{2}\beta I^2 + \gamma \Delta T \tag{13}$$

In the same way, equations for the heat dissipated at the hot sink are:

$$Q_h = \alpha I T_h - \frac{1}{2}\rho E I^2 + \frac{\lambda \Delta T}{E} \tag{14}$$

$$Q_h = \alpha I T_h - \frac{1}{2}\beta I^2 + \gamma \Delta T \tag{15}$$

Hence, from the power balance equations, we obtain:

$$Q_h - Q_c = W$$

$$\alpha I T_h - \frac{1}{2}\beta I^2 + \gamma \Delta T - \alpha I T_c - \frac{1}{2}\beta I^2 - \gamma \Delta T = V \cdot I \tag{16}$$

$$\alpha(T_h - T_c)I + \beta I^2 = V \cdot I$$

Finally, the general expression derived from the balance of power is also similar to Eq. (7) but with one sign changed:

$$V = \alpha \Delta T - \beta I \tag{17}$$

Eq. (17) becomes the most significant expression to estimate the behavior of the module working as an electrical power generator, since it computes the voltage provided by the module as a function of the current and the temperatures.

This expression shows that for a given difference of temperatures, the output voltage decays in a proportional way with the current that the module supplies to an external load, in the same way as any DC source. For open circuit, the voltage attains its maximum value given in this case by $\alpha \Delta T$. However as the value of the current increases the output voltage decreases linearly, β being the slope. Therefore $\alpha \Delta T$ plays the role of the fixed internal voltage of any DC source and β is equivalent to the internal output resistance.

3.2. Estimating Seebeck performance curves

Assuming the same value of the internal parameters of the module: α, β, γ (or α, ρ, λ) for Peltier and Seebeck effects, the performance curves can be estimated from Eq. (17). Therefore, taking the values for α and β from Table 1, Eq. (17) becomes:

$$\begin{aligned} V &= 0.054 \cdot \Delta T - 2.431 \cdot I \quad \text{for } T_h = 27°C \\ V &= 0.055 \cdot \Delta T - 2.769 \cdot I \quad \text{for } T_h = 50°C \end{aligned} \tag{18}$$

Consequently, Eq. (18) is the mathematical model of thermoelectric module Marlow DT 12-6 when used as an electric generator, and the characteristic V–I curve is represented in Fig. 3.

For lower values of the temperature at the hot sink, it is more accurate to use the set of parameters found in the first column of Table 1, which were obtained for $T_h = 27$ °C, while for higher values of the temperature it is more accurate to use the second set of parameters. However, the sensitivity to T_h is not as remarkable as to ΔT. In Fig. 3 thez set of response curves are represented for $T_h = 27$ °C and $T_h = 50$ °C simultaneously. The largest difference can be found for high currents, and it is a consequence of larger β for higher temperatures.

Comparing the performance of this module with one module specifically designed for thermoelectric generation, it can be concluded that Peltier modules produce higher voltage and lower current for the same working condition. For example, for $\Delta T = 40$ °C the Peltier module generates up to 2.25 V ($\alpha \Delta T$) or up to 0.8 A $\left(\frac{\alpha \Delta T}{\beta}\right)$, with a maximum power of less than 0.5 W. On the other hand, the module HiZ-14 with a bigger size (63.9×63.9 mm) but less TE elements (49) generates up to 0.75 V or up to 4.5 A, with a maximum power of 1.75 W, as tested in [3]. As expected, the TEG module can generate more electrical power for the same temperature conditions; however it delivers that power at a lower voltage.
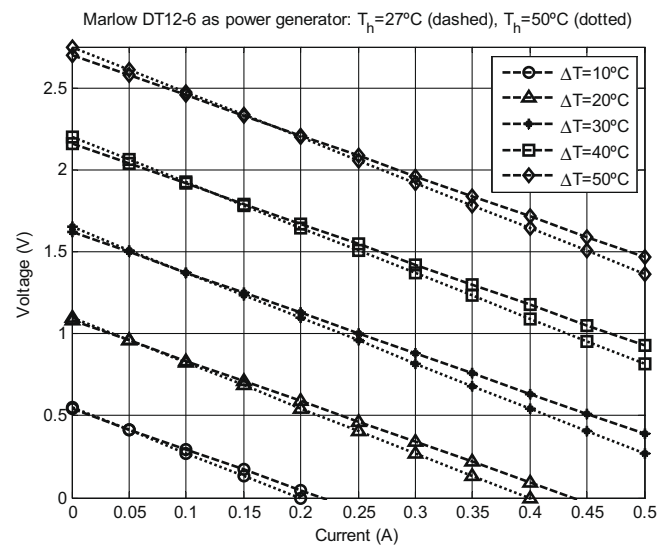

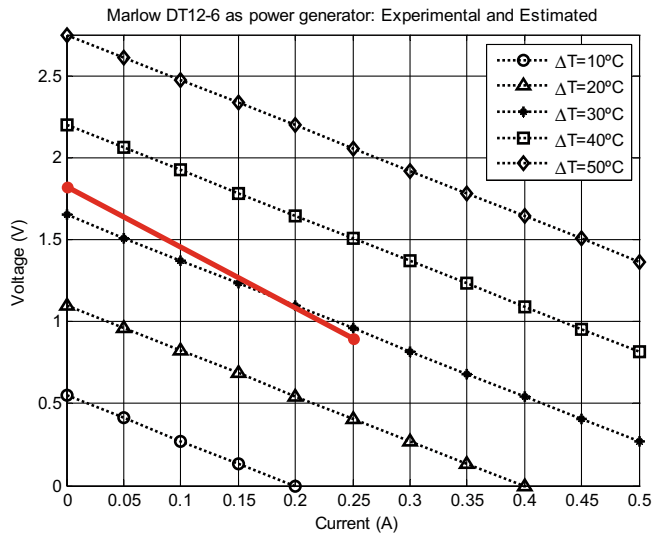**Fig. 3.** Estimated response for $T_h = 27$ °C and $T_h = 50$ °C.

Fig. 4. Experimental and estimated response for ΔT = 35 °C.

4. Experimental verification

In order to validate the proposed analytical procedure, several experimental tests have been carried out using a module Marlow DT 12-6. The module was mounted in between cupper plates and was heated on one side, using a controlled gas burner, while the other side was refrigerated using a water heat exchanger. Once the steady state was reached and a given temperature difference becomes stable, two working points of current and voltage were measured by changing the load resistance of the circuit.

The experimental results obtained for a measured ΔT of 35 °C are shown in Fig. 4 in thick line compared to the analytical estimations. Although the experimental values are slightly higher than the analytical line corresponding to ΔT = 30 °C, the actual internal voltage is a bit lower than the value predicted by the model. The experimental voltage decreases with the current faster than expected so the actual internal resistance is somewhat higher. The internal resistances obtained analytically and experimentally, are in the same order of magnitude as typical values obtained in

previous experiments with TE modules, which are significantly higher than those in other DC sources [4].

5. Conclusions

This paper describes an analytical procedure to estimate internal parameters of commercial thermoelectric modules from the performance curves provided by the manufacturers. After these parameters have been obtained, it is possible to estimate the response of the module under any working condition. Even if the module was designed for cooling applications and only cooling performance curves are provided, the proposed methodology allows researchers to estimate the behavior when the module is used for power generation applications. While the experimental measurements do not perfectly match the results estimated analytically, they can be considered rather precise considering the difficulties for attaining accurate experimental measurements in a standard laboratory. The error of the analytical procedure estimating TEG behavior can be considered on the same level as the error of experimental measurements that try to reproduce the performance curves provided by the manufacturer.

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